

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Introduction:

Conducting materials play a vital role in Engineering. It is very essential to know the electrical properties of materials for specific application of the materials. The properties of metals such as electrical conduction, thermal conduction, specific heat etc., are due to the free electrons or conduction electrons in metals. The first theory to explain the electrical conductivity of metals is Classical free electron theory and it was proposed by Drude in the year 1900 and later developed and refined by Lorentz. Hence classical free electron theory is known as Drude-Lorentz theory.

Assumptions of Classical Free Electron Theory:

1. A metal is imagined as a three dimensional ordered network of positive ions with the outermost electrons of the metallic atoms freely moving about the solid. The electric current in a metal, due to an applied field, is a consequence of drift velocity of the free electrons in a direction opposite to the direction of the field.
2. The free electrons are treated as equivalent to gas molecules and thus assumed to obey the laws of kinetic theory of gases. As per kinetic theory of gases, in the absence of the field the energy associated with each electron at a temperature T is $\frac{3}{2}kT$, where k is Boltzmann constant. It is related to the kinetic energy through the relation

$$\frac{3}{2}kT = \frac{1}{2}mv_{th}^2$$

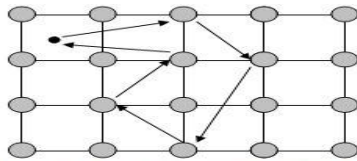
Where v_{th} is the thermal velocity of the electrons.

3. The electric potential due to the ionic core (lattice) is taken to be essentially constant throughout the metal.
4. The attraction between the free electrons and the lattice ions and the repulsion between the electrons are considered insignificant.

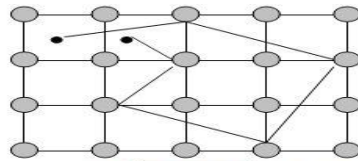
UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Drift Velocity

Initially the electrons in the metal which are in thermal equilibrium will move in random directions and often collide with ions with no net displacement. When electric field is applied, the equilibrium condition is disturbed and there will be net displacement in randomly moving free electron's positions, with time in a direction opposite to the direction of the field. This displacement per unit time is called **drift velocity** which will be constant for the free electrons in the steady state. This accounts for the current in the direction of the field.



Random motion without the electric field. In all probability the electron comes back to a position close to the earlier position if not the same position



Direction of the electric field
Random motion in the presence of the electric field.
Notice the drift of the electron

If 'E' is the electric field applied to the metal, ' τ ' is mean collision time, then drift velocity for conduction electron in a metal is given by

$$v_d = \frac{eE}{m} \tau$$

Where 'e' and 'm' are charge and mass of electron respectively.

Current density (J):

It is the current per unit area of cross section of an imaginary plane held normal to the direction of current in a current carrying conductor.

i.e. $J = I/A$ where A is the area of cross section.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Electric Field (E):

Electric field across homogeneous conductor is defined as the potential drop per unit length of the conductor.

If 'L' is the length of a conductor of uniform cross section and uniform material composition and 'V' is the potential difference between its two ends, then electric field 'E' is given by

$$E = V/L$$

Mean Free Path (λ):

It is the average distance travelled by the conduction electrons between successive collisions with lattice ions.

Mean Collision Time (τ):

It is the average time that elapses between two consecutive collisions of an electron with the lattice ions.

Relation between v , τ and λ :

If 'v' is the total velocity of the electrons, then the mean collision time ' τ ' is given by

$$\tau = \frac{\lambda}{v}$$

Resistivity (ρ):

For a material of uniform cross section, the resistance 'R' is directly proportional to length 'L' and inversely proportional to area of cross section 'A'

i.e.
$$R \propto \frac{L}{A} \quad \therefore R = \rho \left(\frac{L}{A} \right)$$

Where ' ρ ' is called resistivity. It is the property of the material and gives the measure of opposition offered by the material during the current flow in it.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$\therefore \rho = \frac{RA}{L}$$

Conductivity (σ):

It is reciprocal of resistivity. It is a physical property that characterizes conducting ability of a material.

$$\sigma = \frac{1}{\rho} = \frac{L}{RA}$$

Relation between J , σ and E :

From ohms law

$$V=IR = I \cdot \frac{\rho l}{A}$$

$$\frac{I}{A} = \frac{V}{l} \cdot \frac{1}{\rho}$$

$$\frac{I}{A} = J \text{ and } \frac{1}{\rho} = \sigma$$

$$J = \sigma E$$

Expression for electric current in a conductor: (I)

$$I = nev_d A$$

n - Number of free electrons in unit volume of the conductor

v_d - Drift velocity of electrons

A - Area of cross section of the conductor

e - Charge of an electron

Expression for Conductivity:

The current through the conductor is given by

$$I = neAv_d$$

We know that drift velocity (v_d) is given by

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$v_d = \frac{eE\tau}{m}$$

Substituting for v_d in the above equation

$$I = \frac{ne^2\tau}{m} AE$$

$$I = \frac{ne^2\tau A}{m} \left(\frac{V}{L}\right) \quad \text{where 'L' is the length of the conductor}$$

Rearranging the terms of the above equation

$$\frac{m}{ne^2\tau} = \frac{A}{L} \left(\frac{V}{I}\right)$$

$$\frac{m}{ne^2\tau} = \frac{A}{L} R = \rho$$

$$\frac{1}{\rho} = \sigma, \text{Therefore } \sigma = \frac{ne^2\tau}{m}$$

Mobility of electrons:

Mobility of electrons (μ) is defined as the magnitude of drift velocity acquired by the electron in unit field.

$$\text{i.e. } \mu = \frac{v_d}{E} = \frac{1}{E} \left(\frac{eE\tau}{m} \right) = \frac{e\tau}{m}$$

Failures of classical free electron theory:

Although electrical and thermal conductivity in metals can be explained successfully through classical free electron theory, it failed to account for many other experimental facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

1. The molar specific heat of a gas at constant volume is $C_v = \frac{3}{2}R$, where R is a universal constant. But the experimental value of electronic specific heat is $C_v = 10^{-4}RT$ which the classical theory could not explain. Also the experimental value shows that the electronic specific heat is temperature dependent, whereas the classical free electron theory says that it is temperature independent.
2. The electrical conductivity of a metal is inversely proportional to temperature. According to classical free electron theory, electrical conductivity is inversely proportional to the square root of temperature, i.e. $\sigma \propto \frac{1}{\sqrt{T}}$.
3. Electrical conductivity is given as $\sigma = \frac{ne^2\tau}{m}$
According to classical electron theory electrical conductivity is directly proportional to the electron concentration. But monovalent metals like copper found to have high electrical conductivity than the divalent & trivalent metals like Zinc and Aluminium. Hence CFET fails to explain the observation.
4. Though metals are expected to exhibit negative Hall co-efficient since the charge carriers in them are electrons, some metals like zinc have positive Hall co-efficient. The free electron theory could not explain the positive Hall co-efficient of metals.

Assumptions of quantum free electron theory:

The main assumptions of quantum free electron theory are

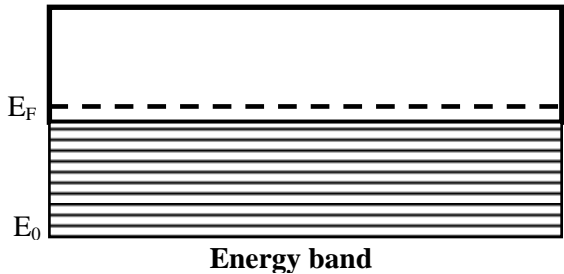
1. The energy values of free electrons are quantized. The allowed energy values are realized in terms of a set of energy levels.
2. The distribution of electrons in the various allowed energy levels, follows Pauli's Exclusion Principle.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

3. Distribution of electrons in energy states obey Fermi-Dirac statistics.
4. The free electrons travel in a constant potential inside the metal but stay confined within its boundaries.
5. The attraction between the free electrons and lattice ions, the repulsion between the electrons themselves are ignored.

Fermi level and Fermi energy:

If we assume the number of electrons per unit volume as n_e then these electrons should be accommodated in the various energy levels. At absolute zero temperature, the electrons occupy the lowest available energy levels. The highest occupied level in metals at zero Kelvin is called as the **Fermi level** and the corresponding energy value of that level is called as the **Fermi energy**, it is denoted by E_F . Thus at 0K all levels up to the Fermi level are occupied while the levels above it are vacant.



The dotted level is the Fermi level. Levels from E_0 up to E_F are occupied while levels above E_F are empty.

Fermi-Dirac statistics:

Under thermal equilibrium the free electrons are distributed in various available energy states. The distribution of electrons among the energy levels follows statistical rule known as Fermi-Dirac statistics.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Fermi-Dirac statistics is applicable to fermions. Fermions are indistinguishable particles with half integral spin. Since electron has half spin they obey Fermi-Dirac statistics and they are called Fermions.

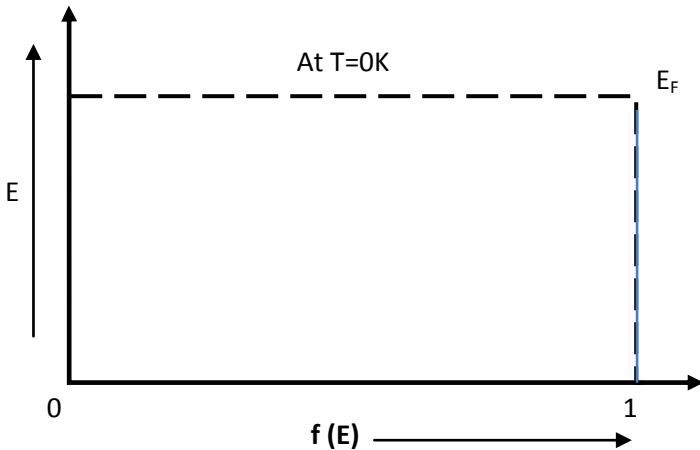
Fermi factor represents the probability that a quantum state with energy E is occupied by an electron, is given by Fermi-Dirac distribution function,

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Where k is the Boltzmann's constant, T is the temperature in Kelvin, E is the energy and E_F is the Fermi energy.

Dependence of Fermi factor on temperature:

The dependence of Fermi factor on temperature at $T=0K$ is given in the figure.



UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Case 1: the probability of occupation for $E < E_F$ at $T = 0K$

Substituting the value of $T = 0K$ in the Fermi function we get

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)+1}} = \frac{1}{e^{-\infty} + 1} = \frac{1}{0+1} = 1$$

$f(E)=1$ implies that all the levels below E_F are occupied by electrons.

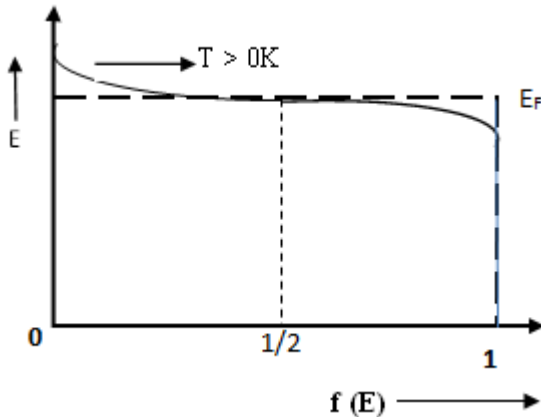
Case 2: the probability of occupation for $E > E_F$ at $T = 0K$.

Substituting the value of $T = 0K$ in the Fermi function, we get

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)+1}} = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} = 0.$$

This shows that all energy levels above E_F are vacant.

Case 3: probability of occupation at temperature $> 0K$



1. At ordinary temperatures, though the value of probability is 1 for $E \ll E_F$, it starts decreasing from 1 as the values of E become closer to E_F .
2. The value of $f(E)$ become $1/2$ at $E=E_F$. This is because at $E = E_F$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)+1}} = \frac{1}{e^0 + 1} = \frac{1}{1+1} = \frac{1}{2}$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

3. For values just beyond E_F , $f(E) > 0$
4. Further above $E > E_F$, the probability value falls off to zero rapidly.

It implies that the probability of occupancy of Fermi level at any temperature other than 0K is 0.5

Hence Fermi level is defined as the energy level at which the probability of electron occupancy is half. Also, Fermi energy, E_F is the average energy possessed by the electrons which participate in conduction process in conductors at temperatures above 0K.

Density of states $g(E)$:

The permitted energy levels for electrons in a solid will be in terms of bands. Each energy band spread over an energy range of few eV. The number of energy levels in each band will be extremely large and hence the energy values appear to be virtually continuous over the band spread. Each energy level consists of two states and each state accommodates only one electron. Therefore, energy level can be occupied by two electrons only, having opposite directions of spin. The exact dependence of density of energy states on the energy is realized through a function denoted as $g(E)$ and is known as density of states function.

It is defined as, the number of available states per unit volume per unit energy interval.

The number of states lying in the range of energies between E and $E+dE$ is given by

$$g(E)dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE.$$

Where E is the kinetic energy of the electron in the energy level E .

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Carrier concentration in metals and Fermi energy at 0K

Number of free electrons /unit volume which possess energy in the range E and E+dE is given by $N(E) dE = g(E) \times dE \times f(E)$

The number of free electrons/unit volume of the material, i.e., n is equal to the total number of electrons that are distributed in various energy levels upto E_F . Thus we have

$$n = \int_{E=0}^{E_F} N(E) dE \qquad n = \int_{E=0}^{E_F} g(E) f(E) dE$$

But, $f(E) = 1$, at $T = 0K$

$$n = \int_{E=0}^{E_F} g(E) dE$$

$g(E) dE$ is given by, $g(E) dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE$

$$n = \frac{4\pi}{h^3} (2m)^{3/2} \int_{E=0}^{E_F} E^{1/2} dE$$

$$n = \frac{4\pi}{h^3} (2m)^{3/2} \frac{2}{3} (E_F)^{3/2}$$

$$n = \frac{8\sqrt{2}}{h^3} \pi m^{3/2} \times \frac{2}{3} (E_F)^{3/2}$$

$$n = \frac{8\pi}{3h^3} (2m)^{3/2} (E_F)^{3/2}$$

This is the equation of concentration of electrons in a metal at 0K.

Expression for the Fermi energy at 0K is given by $E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi}\right)^{2/3}$

$$E_F = Bn^{2/3}$$

Where $B = \left(\frac{h^2}{8m}\right) \left(\frac{3}{\pi}\right)^{2/3}$ is a constant $= 5.85 \times 10^{-38} J$.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Success of Quantum Free Electron theory

1. The theory could successfully explain the specific heat capacity of metals.
2. It could also explain temperature dependence of electrical conductivity.
3. It explained the dependence of electrical conductivity on electron concentration.
4. It also explained photoelectric effect, Compton effect, Black body radiation, Zeeman effect etc.,

HALL EFFECT:

When a transverse magnetic field 'B' is applied perpendicular to current carrying conductor, a potential difference is developed across the specimen in a direction perpendicular to both current and the magnetic field. This phenomenon is called the Hall effect. The Voltage so developed is called Hall voltage.

Hall effect helps to i) determine the sign of charge carrier in the material 2) determine the charge carrier concentration and iii) determine the mobility of charge carrier, if conductivity of material is known. Hall effect measurement showed that the negative charge carriers, that is the electrons are responsible for conduction in metal and it also showed that there exists two types of charge carriers in a semiconductor.

To explain Hall Effect in **metals** (conductor), consider a rectangular plate of a metal having, width w and thickness t . When potential difference is applied across its ends, a current 'I' flows through it along x direction that is opposite to the direction of flow of electrons. The current passing through the metal is given by

$$\mathbf{I = nAev_d} \quad (1)$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

where n - concentration of electrons

A - Area of cross section of end face

e - charge on the electron

v_d - drift velocity of electron

n – concentration of electrons

$$\text{Therefore } v_d = \frac{I}{nAe} = \frac{I}{nwt e} \quad (2)$$

where $A=wt$ (w is the width of the sample and t is the thickness of the sample)

Any plane perpendicular to current flow direction is an equipotential surface. Therefore potential difference between front and rear faces is zero. If magnetic field is applied normal to crystal surface and also to the current flow, a transverse potential difference is produced between the faces F & F' . It is called Hall voltage V_H .

In the absence of magnetic field B , the charge carriers move in a direction parallel to faces F & F' . On the application of magnetic field B , the Lorentz force comes into existence and this force deflects the electrons sideways. The magnitude of this force is given by

$$F_L = e B v_d$$

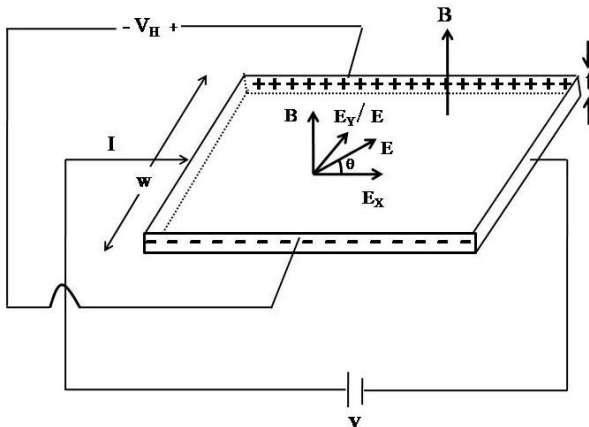


Fig. Hall effect in conductors

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

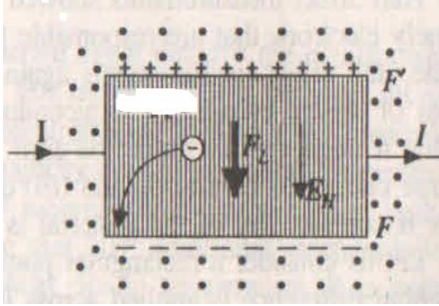


Fig. Direction of magnetic force and Hall field in a conductor

In the absence of magnetic field B , the charge carriers move in a direction parallel to faces F & F' as shown in the above figure. On the application of magnetic field B , the Lorentz force comes into existence and this force deflects the electrons sideways due to the magnetic force F_L . The magnitude of this force is given by

$$F_L = e B v_d$$

Because of this force, electrons will move towards the front face F and pile up there. Initially the material is electrically neutral everywhere. However, as electrons pile up on the front side, a corresponding equivalent positive charge is left on the rear face F' . As a result an electric field is produced across F & F' . The direction of electric field (E_H) will be from rear face to front face. A condition of equilibrium is reached when force (F_H) due to transverse electric field E_H , balances the Lorentz force (F_L). The transverse electric field E_H is known as Hall field.

At equilibrium condition

$$F_L = F_H \quad (3)$$

$$F_H = e E_H = e (V_H/w) \quad (4)$$

Substituting for F_L and F_H in eqn (3) we get

$$eBv_d = eV_H/w \quad (5)$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Substituting for v_d from equation (2) in equation (5)

$$\frac{BI}{nwt} = \frac{V_H}{w} \quad (6)$$

$$\text{Hall voltage } V_H = \frac{BI}{net} \quad (7)$$

Reciprocal of Carrier charge density is called **Hall co-efficient R_H**

$$R_H = 1/ne$$

Equation (6) can be written as

$$\frac{1}{ne} = \left(\frac{V_H}{B}\right) \frac{t}{I}$$
$$R_H = \frac{V_H t}{BI}$$

The Hall voltage can be measured with a voltmeter with the direction of magnetic field & current depicted in fig, the sign of Hall voltage is negative.

The charge carrier concentration is given by

$$n = -\frac{1}{R_H e} = -\frac{BI}{V_H t}$$

BAND THEORY OF SOLIDS:

The energy band structure of a solid determines whether it is a conductor, an insulator, or a semiconductor

The electron of an isolated atom has certain definite energies such as 1s, 2s, 3p, 3s, etc. Between two consecutive allowed values of energy there is forbidden gap. As we bring together large number of identical atoms to form a solid, significant changes take place in the energy levels. The energy levels of each atom will interact with the other identical atoms. The wave functions of each atom will overlap and as a result the energy levels of each atom overlap slightly and

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

split into a number of levels corresponding to the number of atoms. The split energy levels are very close to each other and they form a narrow band known as energy band.

The range of energies possessed by electrons in a solid is known as energy band.

The energy band formed by the energy levels of the valence electrons is called valence band. The energy band immediately above the valence band where the conduction electrons are present is called conduction band.

The separation between the upper level of valence band and the bottom level of conduction band is known as forbidden energy gap, E_g .

The forbidden energy gap is a measure of the bondage of valence electrons to the atom. The greater the energy gap more tightly valence electrons are bound. When energy is supplied, electrons from the valence band jump to the conduction band and thereby the material starts conducting.

SEMICONDUCTORS

Pure semiconductors are the materials having electrical conductivity greater than that of insulators but significantly lower than that of a conductor at room temperature. They have conductivity in the range of 10^{-4} to 10^4 S/m. The interesting feature about semiconductors is that they are bipolar and current is transported by two types of charge carriers of opposite sign namely electrons and holes. The number of carriers can be drastically enhanced by doping the semiconductor with suitable impurities. The doped semiconductor

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

which exhibits higher conductivity is called an extrinsic semiconductor. The conductivity of an extrinsic semiconductor depends on the doping level which is amenable to control. The current transportation in extrinsic semiconductor occurs through two different processes namely drift and diffusion. Pure semiconductors are of relatively less importance whereas extrinsic semiconductors are widely used in fabricating devices. These devices are more generally known as solid-state electronic devices.

INTRINSIC SEMICONDUCTORS

A semiconductor in an extremely pure form is known as an intrinsic semiconductor.

Intrinsic carriers in pure semiconductors

At room temperature in pure semiconductors, a single event of breaking of bonds leads to two carriers; namely electron and hole. The electron and hole are created as a pair & the phenomenon is called electron-hole pair generation. At any temperature T the number of electrons generated will be equal to the number of holes generated. If 'n' denotes number density of electrons in the conduction band & 'p' denotes the number of holes in the 'valence band then $n = p = n_i$ where, ' n_i ' is called intrinsic concentration or the intrinsic density

After the generation, the carriers move independently; the electrons move in the conduction band & the holes move in the valence band. The motion of these two carriers is random in their respective band as long as no external field is applied.

Concept of Effective Mass of the Electron and Holes:

Consider an isolated electron of mass m and charge $-e$ in an electric field of strength E . The electric force acting on it is $-eE$. The

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

electron gets accelerated, then $-eE = ma$. However, an electron within a crystal is in a periodic potential due to positive ion cores. The neighbouring ions and electrons in the crystal do exert some force on the electron in a crystal. Then $ma = -eE + \text{force due to neighbouring ions and electrons}$. Since the latter force is not known quantitatively, we can write the above equation as $m_e^*a = -eE$ or $m_e^* = -eE/a$ where m_e^* is called the effective mass of the electron within the crystal. Thus it is inferred that the effective mass of an electron depends on its location in the energy band. Electrons near the bottom of the conduction band have an effective mass which is almost equal to the effective mass of a free electron. Electrons near the bottom of the valence band have negative effective mass. The removal of an electron with a negative effective mass is identical to creating a particle of positive mass. Thus hole is given the status of particle with positive effective mass m_h^* .

Carrier concentration in intrinsic semiconductor

The actual number of electron in the conduction band is given by

$$n = \int_{E_c}^{\text{top of the band}} f(E)g_c(E)dE \quad (1)$$

Since F-D function describes the probability of occupancy of energy state. Under thermal equilibrium condition, the electron concentration obtained from eqn. (1) is the equilibrium concentration.

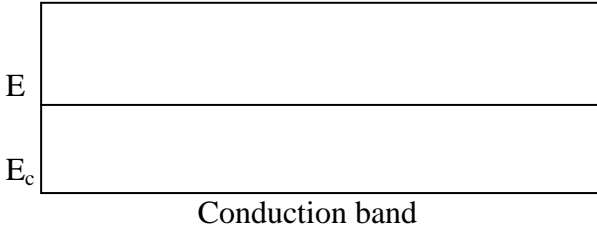
As $f(E)$ rapidly approaches zero for higher energies, the integral in eqn. (1) can be re-written as

$$n = \int_{E_c}^{\infty} f(E)g_c(E)dE$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$g_c(E) = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} (E)^{\frac{1}{2}} dE$$

Where, E is the kinetic energy of the electron.



In the above fig. the bottom edge of conduction band E_C corresponds to the potential energy of an electron at rest in conduction band. Therefore the quantity $(E - E_C)$ represents the kinetic energy of conduction level electron at high energy level.

$$g_c(E) = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} dE \quad (2)$$

$$n = \int_{E_c}^{\infty} \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \frac{(E - E_c)^{\frac{1}{2}}}{1 + \exp\left[\frac{(E - E_F)}{kT}\right]} dE \quad (3)$$

As $E > E_F$: $e^{\frac{E - E_F}{kT}} \gg 1 : 1 + e^{\frac{E - E_F}{kT}} \cong e^{\frac{E - E_F}{kT}}$

Therefore $\frac{1}{1 + e^{\frac{E - E_F}{kT}}} \cong e^{\frac{-(E - E_F)}{kT}}$

Using this eqn in eqn. (3) we get

$$n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} e^{\frac{-(E - E_F)}{kT}} dE$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Add and subtract E_c to the exponential term in the above equation.

$$n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{kT}} \int_{E_c}^{\infty} (E - E_c)^{\frac{1}{2}} e^{\frac{-(E - E_c)}{kT}} dE$$

Let $E - E_c = x$ then $dx = dE$

Lower limit when $E = E_c$ $x = E_c - E_c = 0$

Upper limit when $E = \infty$ $x = \infty - E_c = \infty$

Therefore
$$n = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} e^{\frac{(E_F - E_c)}{kT}} \int_0^{\infty} x^{\frac{1}{2}} e^{-ax} dx \quad (4)$$

The integral is similar to standard integral.

The solution of eqn.(4) is given by

$$\int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}, \quad \text{where } a = 1/kT$$

$$\therefore n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F - E_c)/kT} \left(\frac{\sqrt{\pi}}{2} (kT)^{3/2} \right)$$

Rearranging the term we get

$$n = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} e^{-(E_c - E_F)/kT} \quad (5)$$

$$\text{Let } N_c = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2}$$

$$\therefore n = N_c e^{-(E_c - E_F)/kT} \quad (6)$$

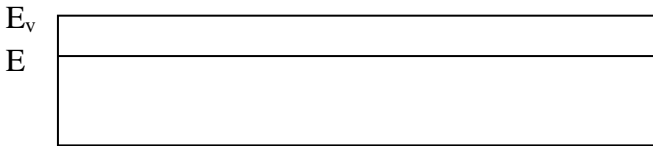
N_c is temperature-dependent material constant known as effective density of states in the conduction band.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Expression for hole concentration in valence band

If $f(E)$ is the probability for occupancy of an energy state at E by an electron, then probability that energy state is vacant is given by $[1 - f(E)]$. Since hole represents the unoccupied state in valence band, the probability for occupancy of state at E by a hole is equal to probability of absence of electron at that level. The hole concentration in valence band is therefore given by

$$p = \int_{\text{bottomband}}^{E_v} [1 - f(E)] g_v(E) dE \quad (7)$$



Solving equation 7 we arrive at hole concentration, $p = N_v e^{\frac{-(E_F - E_v)}{kT}}$

Where, N_v is temperature-dependent material constant known as effective density of states in the valence

band. Let $N_v = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$

Fermi level in intrinsic semiconductor

In an intrinsic semiconductor electron and hole concentrations are equal.

Therefore $n = p$

$$N_c e^{\frac{-(E_C - E_F)}{kT}} = N_v e^{\frac{-(E_F - E_v)}{kT}}$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Taking logarithm on both side and rearranging the term, we get

$$\Rightarrow \frac{-(E_C - E_F)}{kT} = \ln\left(\frac{N_V}{N_C}\right) - \frac{(E_F - E_V)}{kT}$$

Multiplying by kT throughout

$$\Rightarrow -E_C + E_F = kT \ln\left(\frac{N_V}{N_C}\right) - E_F + E_V$$

$$\Rightarrow 2E_F = kT \ln\left(\frac{N_V}{N_C}\right) + E_C + E_V$$

$$\Rightarrow E_F = \left(\frac{E_C + E_V}{2}\right) + \frac{1}{2}kT \ln\left(\frac{N_V}{N_C}\right)$$

Substituting the values of N_V and N_C and after simplification we get

$$E_F = \left(\frac{E_C + E_V}{2}\right) + \frac{3}{4}kT \ln\left(\frac{m_h^*}{m_e^*}\right) \quad (1)$$

As kT is small and the effective mass m_e^* and m_h^* do not differ much, the second term in the eqn. (1) may be ignored.

If $m_e^* = m_h^*$, then we get

$$E_F = \left(\frac{E_C + E_V}{2}\right) \quad (2)$$

we can write eqn. (2) as

$$E_F = \left(\frac{E_C + E_V + E_V - E_V}{2}\right) = \frac{E_C - E_V}{2} + E_V$$

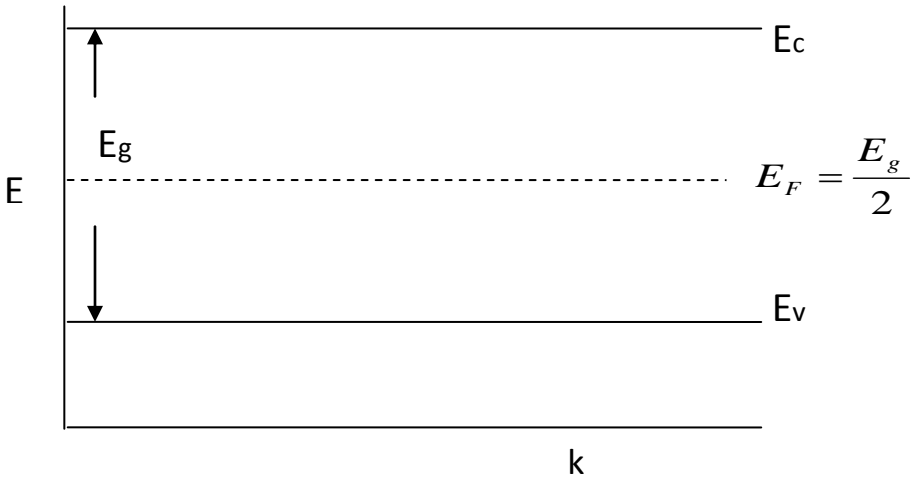
but $E_C - E_V = E_g$

$$\therefore E_F = \frac{E_g}{2} + E_V$$

If top of the valence band E_V is taken as zero level, then $E_F = \frac{E_g}{2}$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Thus Fermi level in the intrinsic semiconductor lies at the centre of the energy gap as shown below:



INTRINSIC DENSITY, n_i

In an intrinsic semiconductor at $T=0K$, the electron concentration in the conduction band is identical to hole concentration in the valence band.

$$n=p=n_i$$

From this, we get

$$\begin{aligned} np &= n_i^2 \\ n_i^2 &= N_C e^{\frac{(E_c - E_F)}{kT}} N_V e^{\frac{(E_F - E_V)}{kT}} \\ &= (N_C N_V) e^{\frac{(E_c - E_V)}{kT}} \end{aligned}$$

But $E_c - E_V = E_g$

$$\begin{aligned} n_i^2 &= (N_C N_V) e^{-E_g / kT} \\ n_i &= (N_C N_V)^{1/2} e^{-E_g / 2kT} \end{aligned}$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Substituting the values of N_c and N_v we get,

$$n_i = 2 \left[\frac{2\pi k}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} T^{3/2} e^{-E_g/2kT}$$

The following important points may be inferred from the above relation

1. The intrinsic density is independent of Fermi level position.
2. The intrinsic density is a function of band-gap E_g , which represents the energy needed to break a bond.
3. The intrinsic density strongly depends on the temperature. The contribution of temperature increase to n_i is mostly due to the exponential term and only to a marginal extent due to the term $T^{3/2}$.

Extrinsic semiconductor

The intrinsic semiconductor has low conductivity which is not amenable to control. However a judicious introduction of impurity atoms in an intrinsic semiconductor produces useful modification of its electrical conductivity. The method of introduction of controlled quantity of impurity into an intrinsic semiconductor is called doping. The impurity added is called dopant. The semiconductor doped with impurity atoms is called extrinsic semiconductor. There are two types of extrinsic semiconductor namely p-type & n-type which are produced depending on the group of impurity atoms.

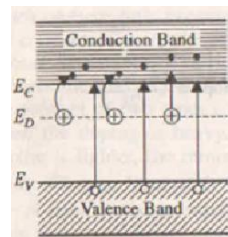
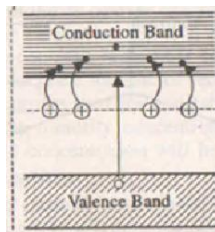
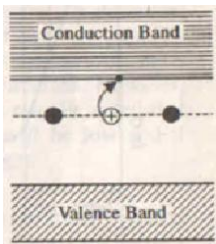
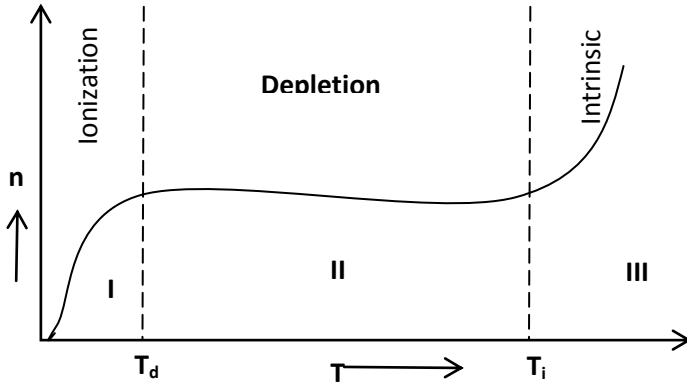
n-type semiconductors are produced when pure semiconductors are doped with pentavalent impurity atoms such Phosphorous, Arsenic etc.

p-type semiconductors are produced when pure semiconductors are doped with trivalent impurity atoms such as Aluminum, Boron etc.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Temperature variation of carrier concentration in extrinsic semiconductor

The dependence of electron concentration on temperature for **n-type** semiconductor is as shown in the figure below.



At 0K the donor levels are filled which means that all the donor electrons are bound to the donor atoms. At low temperature, corresponding to region- I, there is not enough energy to ionize all the donors and not at all enough to break covalent bond. As temperature increases, the donor atoms get ionized and donor atoms go into the conduction band. The region-I is known as ionization region. Occasionally a covalent bond maybe broken out, but number of such events will be insignificantly small. At about 100K all donor atoms are ionized, once all electrons from donor level are excited

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

into conduction band, any further temperature increase does not create additional electrons and the curve levels off. The region II is called depletion region. In the depletion region the electron concentration in the conduction band is nearly identical to the concentration of dopant atoms.

If N_D is donor concentration then

$$n_n = N_D \text{ (depletion region)}$$

Where n_n – electron concentration in n-type

As temperature grows further, electron transitions from valence band to conduction band increases. At high temperature (region-III) the number of electron transition becomes so large that the intrinsic electron concentration exceeds the electron concentration due to donor. This region is therefore called intrinsic region. In intrinsic region, $n_n = n_i$

P type semiconductor

Similarly in p-type semiconductor, the acceptor levels are vacant at 0K & valence band is full. As temperature increases in the ionization region, the electrons from the valence band jump into acceptor level. However, the electrons do not acquire enough energy to jump into conduction band levels. At the temperature T_s , the acceptor levels are saturated with electrons. The region- II lying between T_s (saturation temperature) and T_i is called the saturation region. In case of p-type materials within this temperature interval the hole concentration remains constant as thermal energy is not yet sufficient to cause electron transition from valence band to conduction band. In the saturation region, the hole concentration is equal to the acceptor impurity concentration. Thus $p_p = N_A$

With increase of temperature beyond T_i , electron transition due to intrinsic process commence & hole concentration due to intrinsic process far exceeds that due to impurity atoms.

In region-III, $p_p = n_i$

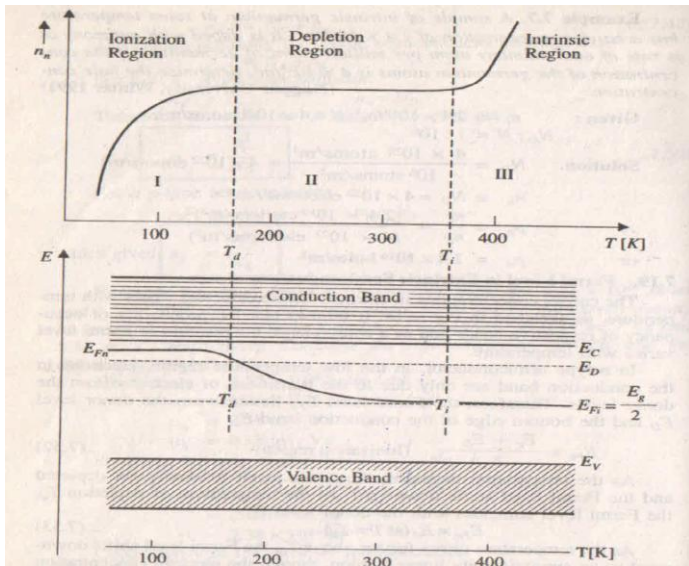
UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Fermi level in extrinsic semiconductor

N-type semiconductor

The carrier concentration in extrinsic semiconductors varies with temperature as discussed earlier. It follows that the probability of occupancy of respective bands & position of Fermi level varies with temperature.

In n-type semiconductor, in low temperature region the electron in the conductor band is only due to the transition of electrons from donor levels. Therefore Fermi level lies between the donor level E_D & the bottom edge of conduction band.



As temperature increases the donor level gradually gets depleted & the Fermi level shifts downward. At the temperature of depletion T_d , the Fermi level coincides with the donor level E_D

$$\text{i.e. } E_{Fn} = E_D.$$

As temperature increases further above T_d , the Fermi level shifts downward approximately in a linear fashion, though electron

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

concentration in the conduction band remains constant. This is in accordance with the relation $E_{F_n} = \frac{E_c + E_D}{2} - \frac{kT}{2} \ln \frac{N_c}{N_D}$.

At temperature T_i , where intrinsic process contributes to electron concentration significantly, the Fermi level approaches the intrinsic value $E_{F_i} = E_g/2$. With further increase in temperature the behaviour of extrinsic semiconductor transitions into that of an intrinsic type & Fermi level stays at E_{F_i} . Thus

$$E_{F_n} = E_{F_i} = E_g/2.$$

P-type semiconductor

In case of p-type semiconductor the Fermi level E_{F_p} rises with increasing temperature from below the acceptor level to intrinsic level E_{F_i} as shown in fig2.

$$\therefore E_{F_p} = \frac{E_A + E_v}{2} \quad (\text{ionization region})$$

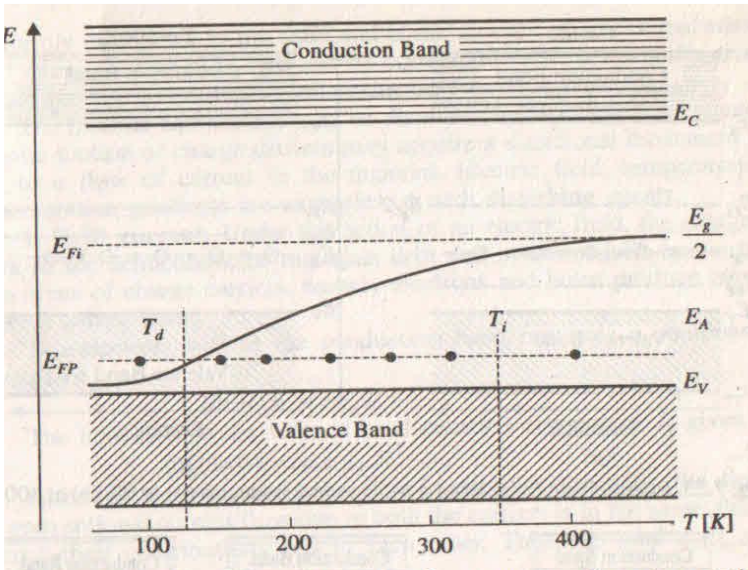
As temperature increases further above T_s , the Fermi level shifts downward approximately in linear fashion, though hole concentration in the valence band remains constant. This is in

accordance with the relation $E_{F_p} = \frac{E_v + E_A}{2} + \frac{kT}{2} \ln \frac{N_v}{N_A}$.

$$E_{F_p} = E_A \quad (\text{at } T=T_s)$$

$$\text{And } E_{F_p} = E_g/2.$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS



Effect of variation of impurity concentration: n-type semiconductor

The addition of donor impurity to an intrinsic semiconductor leads to the formation of discrete donor level below the bottom edge of conduction band. At low impurity concentrations the impurity atom are spaced far apart & do not interact with each other. With an increase in the impurity concentration the impurity atom separation in the crystal decreases & they tend to interact. Consequently the donor level also undergoes splitting & form energy band below the conduction band. The larger the doping concentration, the broader is the impurity band & at one stage it overlaps with the conduction band.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

The broadness of donor levels into a band is accompanied by a decrease in the width of forbidden gap & also the upward displacement of Fermi level. The Fermi level moves closer & closer to the conduction band with increasing impurity concentration & finally moves into the conduction band as donor band overlaps the conduction band.

In similar way, in p-type semiconductor, the acceptor level broadens and forms into a band with increasing impurity concentration which ultimately overlaps the valence band. The Fermi level moves down closer to the valence band and finally at high impurity concentration it will shift in to valence band

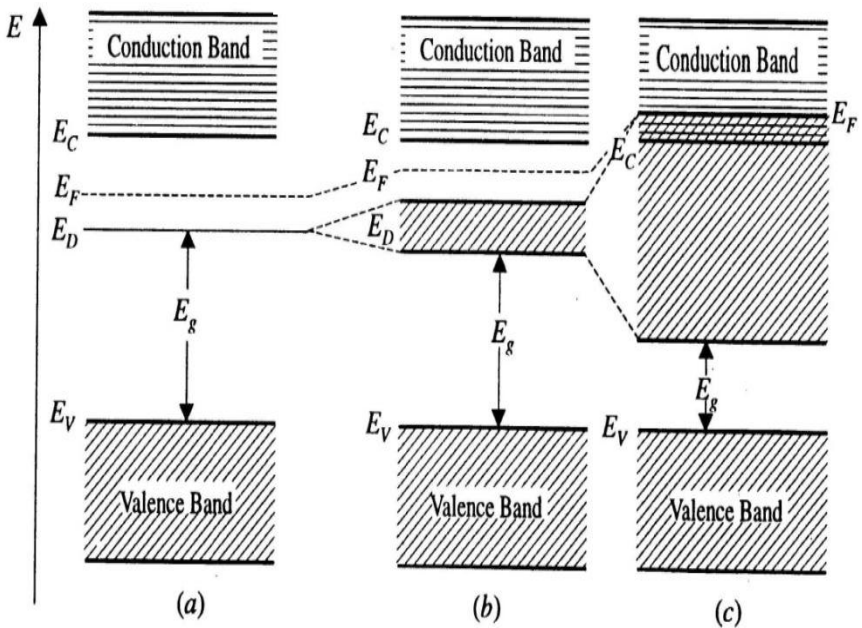


Fig. 7.24. Energy band diagrams of an *n*-type semiconductor at three different levels of doping; (a) low level doping; (b) medium doping; (c) heavy doping.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

HALL EFFECT IN SEMICONDUCTORS:

The Hall effect in metals is similar to that of semiconductors. In case of metals and n -type semiconductors the majority charge carriers are electrons and hence the same derivation holds good for both. In a p-type semiconductor the only difference is that majority charge carriers are **holes**, with positive charge, hence with slight modifications we can obtain the Hall effect parameters following the method used for metals.

Let us consider a rectangular plate of p-type semiconductor. When potential difference is applied across its ends, a current 'I' flows through it along the direction. If holes are majority charge carriers in p-type semiconductors then the current is given by

$$I = pAev_d \quad (1)$$

Where p- concentration of holes

A-Area of cross section of end face

e- charge on the hole

v_d - drift velocity of holes

$$\text{therefore } v_d = I/pAe = I/pwte \quad (2)$$

Any plane perpendicular to current flow direction is an equipotential surface. Therefore potential difference between front and rear faces is zero. If magnetic field is applied normal to crystal surface and also to the current flow, a transverse potential difference is produced between the faces F & F' (fig). It is called Hall voltage V_H .

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

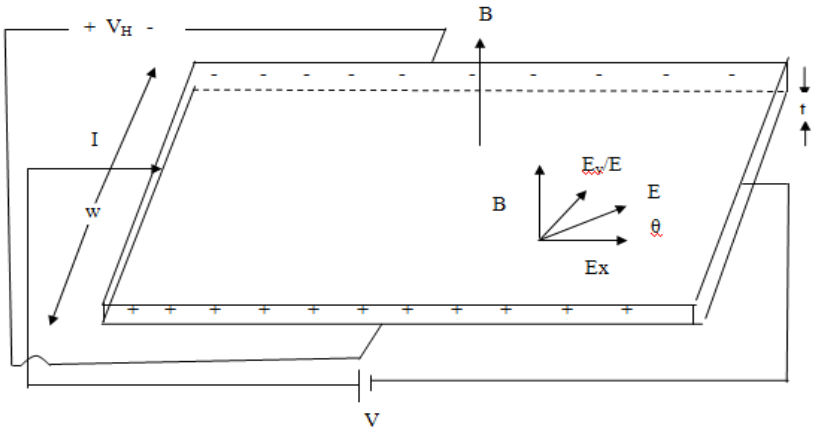


Fig. Hall effect in p type semi conductor

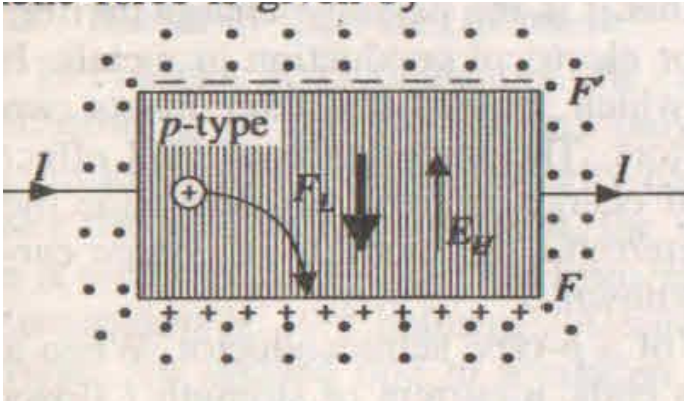


Fig. Direction of magnetic force and Hall field in p-type semiconductor

Before the application of magnetic field B , the holes move in an orderly way parallel to faces F & F' . On the application of magnetic field B , the holes experience a sideways deflection due to the Lorentz force F_L . The magnitude of this force is given by

$$F_L = e B v_d$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Because of this force, holes are deflected towards the front face F and pile up there. Initially the material is electrically neutral everywhere. However, as holes pile up on the front side, a corresponding equivalent negative charge is left on the rare face F' . As a result an electric field is produced across F & F' . The direction of electric field will be from front face to rare face. It is such that it opposes the further pile up of holes on the front face F . A condition of equilibrium is reached when F_H due to transverse electric field E_H balances the Lorentz force. The transverse electric field E_H is known as Hall field.

In equilibrium condition

$$F_L = F_H$$

$$F_H = e E_H = e(V_H/w) \quad (3)$$

Substituting for F_L and F_H in eqn (3) we get

$$eBv_d = eV_H/w \quad (5)$$

Substituting for v_d from equation (2) in equation (5)

$$\frac{BI}{pwte} = \frac{V_H}{w} \quad (6)$$

$$\text{Hall voltage } V_H = \frac{BI}{pet} \quad (7)$$

Reciprocal of Carrier charge density is called **Hall co-efficient R_H**

$$R_H = 1/pe$$

Equation (6) can be written as

$$\frac{1}{pe} = \left(\frac{V_H}{B}\right) \frac{t}{I} \quad R_H = \frac{V_H t}{BI}$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

The Hall voltage is a real voltage & can be measured with a voltmeter with the direction of magnetic field & current depicted in this fig, the sign of Hall voltage is +ve.

For n-type semiconductor Hall voltage will be -ve, when the direction of current is same as in the fig. Therefore by knowing the sign of Hall voltage the type of semiconductor & the sign of the majority charge carriers will be known.

The carrier concentration is given by

$$p = \frac{1}{R_H e} = \frac{BI}{V_H t e}$$

In case of n-type semiconductor

$$n = -\frac{1}{R_H e} = -\frac{BI}{V_H t e}$$

DIELECTRICS

Materials such as glass, ceramics, polymers and paper are non-conducting materials. They prevent the flow of current through them, therefore they can be used for insulation purposes. When the main function of non-conducting materials is to provide electrical insulation they are called **Insulators**.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

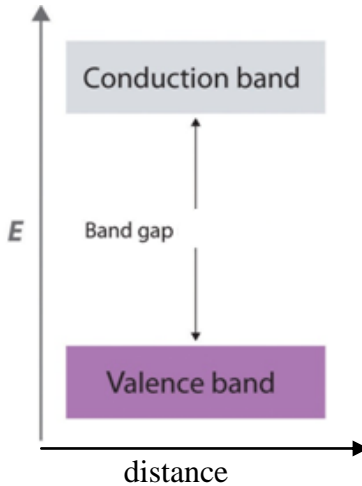


Figure 1: Band diagram of an insulator

When non-conducting materials are placed in an electric field, they undergo appreciable changes as a result of which they act as stores of electric charges. When charge storage is the main function, the insulating materials are called **Dielectrics**. For a material to be a good dielectric, it must be an insulator. Hence any insulator is a dielectric.

The forbidden energy gap (E_g) between the valence band and conduction band is very large (fig.1) in dielectrics and excitation of electrons from valence band to conduction band is not possible under ordinary conditions. Therefore conduction cannot occur in a dielectric. Even if the dielectric contains impurities, extrinsic conduction cannot occur as observed in case of extrinsic semiconductors. The resistivity of an ideal dielectric is infinity, in practise dielectrics conduct electric current to a negligible extent and their resistivities range from 10^{10} to $10^{20}\Omega\text{m}$.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Electric dipole and dipole moment

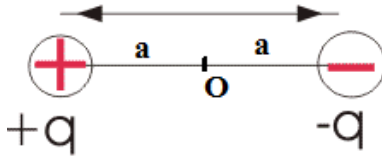


Figure 2: Electric dipole

A pair of equal and opposite charges separated by a small distance is called an **electric dipole** and the product of the magnitude of one of the charges and the distance of their separation is called **dipole moment**(μ).

Consider two charges $-q$ and $+q$ with a distance of separation is $2a$ where a is the distance from the centre of dipole to one of the charge as shown in figure 2. The dipole moment for this arrangement is given by

$$\mu = (2a)q \dots \dots \dots (1)$$

Polar and Non-polar dielectrics:

A dielectric material doesn't possess any free electrons. All the electrons are bound very strongly to the respective nuclei of the atoms of the parent molecules. Each molecule consists of equal number of positive and negative charges. All the positive charges are concentrated in the nuclei, which are surrounded by electron clouds in which all the negative charges are distributed.

If in the molecules of some dielectric materials, the effective centre of the negative charge distribution coincides with the effective centre of the positive charge distribution such materials are called **non-polar dielectrics**. eg Hydrogen, carbondioxide etc.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

In some dielectric materials, the effective centres of the negative and positive charges in the molecules do not coincide with each other in the absence of an external electric field. Each molecule behaves like a **permanent dipole** and the materials comprising of such dipoles are called **polar dielectrics**. eg HCl, H₂O etc.

Polarization

If a dielectric is placed in an electric field of strength 'E', the electron cloud will be displaced in the direction opposite to 'E' by a distance 'd' with respect to the nucleus. The centres of gravity of positive and negative charges in the atom no more coincide.

The atom is equivalent to the system of two charges, $q=Ze$ of equal magnitude but opposite in sign separated by a distance 'd' and the atom behaves like a dipole and it is called **induced dipole**. The atom is said to be **polarized**.

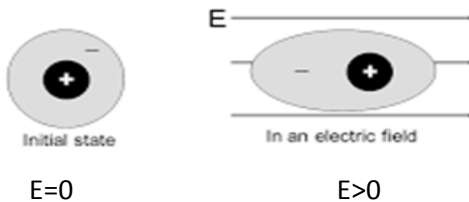


Figure 5

The induced dipole sets up its own electric field which is opposite in direction to the external field.

The dipole moment μ is a vector, directed along the axis of the dipole from the negative charge to the positive charge.

When the molecule is polarized, restoring forces due to coulomb attraction come into play which tends to pull the displaced

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

charges together. The charges separate until the restoring force balances the force due to the electric field.

The induced dipole moment is proportional to the field strength. The larger the field, greater the displacement of charges and hence larger the induced dipole moment. The induced dipole moment is given by

$$\mu = \alpha E$$

α is the **polarizability** of the molecule. It characterizes the capacity of electric charges in a molecule to suffer displacement in an electric field. The unit of polarizability is Fm^2 . The induced dipole moment vanishes as soon as the electric field is switched off.

Dipole in an electric field

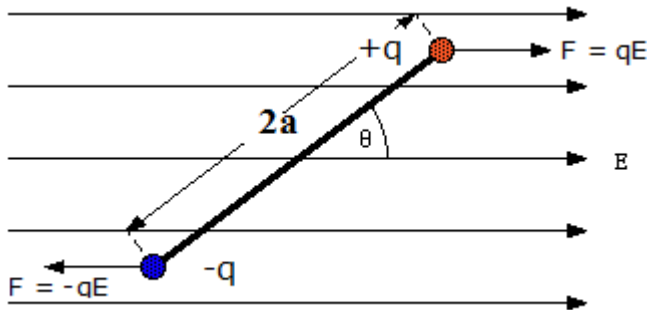


Figure 6: An electric dipole in a uniform electric field

When a polar molecule is placed in a uniform electric field 'E' (figure 6), the field exerts a force $+qE$ on the charge $+q$ and $-qE$ on $-q$. The net force on the dipole is zero since the two forces acting on it are equal and opposite to each other. Therefore, there is no translational force on the dipole in the uniform field. The two forces are anti parallel and separated by perpendicular distance, hence

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

constitutes a couple, which tends to rotate the dipole. The torque on the dipole is given by

$$\tau = qE2a\sin\theta = \mu E\sin\theta \quad (q2a=\mu)$$

Displacement of positive and negative charges in the molecules of a dielectric under the action of applied electric field leading to the development of dipole moment is known as dielectric **Polarization**.

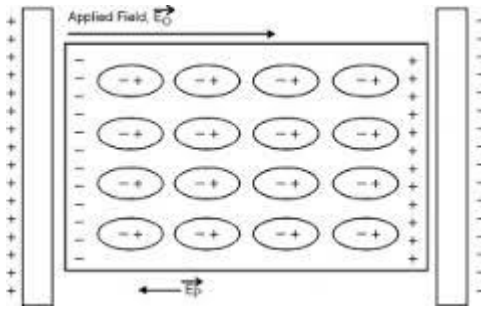


Figure 7

Consider an electrically neutral slab inserted between the plates of a parallel plate capacitor as shown in figure 7. Dielectric is imagined to be divided into large number of identical cells of volume dv . Under the action of external electric field, charges are induced in each cell and each cell acquires a dipole moment $d\mu$.

Then intensity of polarization "P" is defined as the total dipole moment per unit volume of the material.

$$P = \sum \frac{d\mu}{dv} = \frac{\mu}{v}$$

Dielectric constant

For isotropic materials the electric flux density E and the electric induction (or electric displacement) D are related by the equation

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$D = \epsilon_0 \epsilon_r E$$

where $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$, is the dielectric constant of vacuum and ϵ_r is the dielectric constant or relative permittivity for the material. It has no units.

Dielectric susceptibility

The magnitude of polarization is directly proportional to the intensity of the electric field.

Thus, $P = \chi \epsilon_0 E$ (for linear dielectrics)

χ (chi) is the proportionality constant and is called the dielectric susceptibility of the material. It characterizes the ease with which the dielectric material can be influenced by an external field. P is a measure of the polarization produced in the material per unit electric field.

Relation between ϵ_r and χ

In order to describe the combined effects of the applied electric field E and electric polarization P , an auxiliary vector D called **Electric displacement vector** is introduced. $D = \epsilon_0 E + P$

Substituting for $P = \chi \epsilon_0 E$ in the above equation

$$D = \epsilon_0 E + \chi \epsilon_0 E$$

$$D = (1 + \chi) \epsilon_0 E$$

$$D = \epsilon_0 \epsilon_r E$$

$$\text{where } \epsilon_r = 1 + \chi$$

ϵ_0 is the absolute permittivity of the free space and ϵ_r is the relative permittivity or the dielectric constant of the material.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Types of Polarization

i) Electronic or Atomic Polarization:

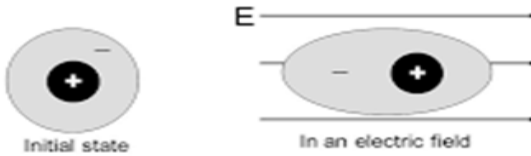


Figure 8

This is the polarization that results from the displacement of electron clouds of atoms or molecules with respect to the heavy fixed nuclei to a distance that is less than the dimensions of atoms or molecules (figure 8). This polarization sets in over a very short period of time, of the order of 10^{-14} - 10^{-15} s. It is independent of temperature.

The polarization is given by $P_e = N\alpha_e E$ (1)

where N is the number of atoms/unit volume, α_e is electronic polarizability.

$$\text{We have } P = \chi \epsilon_0 E \text{ or } \chi = \frac{P}{\epsilon_0 E} \dots(2)$$

$$\text{Dielectric constant } \epsilon_r = 1 + \chi \dots(3)$$

Substituting eqn. (2) in eqn. (3)

$$\epsilon_r = 1 + \frac{P_e}{\epsilon_0 E} \dots(4)$$

Substituting for P_e from eqn (1) in eqn (4)

$$\epsilon_r = 1 + \frac{N\alpha_e E}{\epsilon_0 E}$$

$$\epsilon_r = 1 + \frac{N\alpha_e}{\epsilon_0} \dots\dots\dots (5) \text{ or } \alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N}$$

ϵ_r is the dielectric constant of a non polar gaseous dielectric. The above equation indicates that the dielectric constant depends on the polarizability of a molecule and the number of molecules in a unit volume of the dielectric.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

ii) Ionic Polarization:

Ionic polarization occurs in ionic crystals. It is brought about by the elastic displacement of positive and negative ions from their equilibrium position. Eg. Sodium chloride crystal.

A NaCl molecule consists of Na^+ ion bound to Cl^- ion through ionic bond. If the interatomic distance is 'd', the molecule exhibits an intrinsic dipole moment equal to "qd" where q is the charge of the electron and d is the distance of separation.

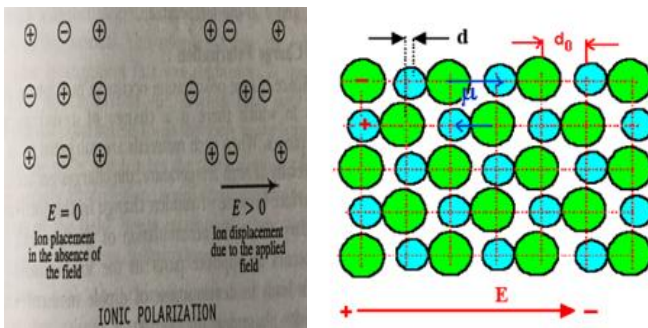


Figure 9

When ionic solids are subjected to an external electric field, the adjacent ions of opposite sign undergo displacement (figure 9). The displacement causes an increase or decrease in the distance of separation between the atoms depending upon the location of the ion pair in the lattice. This polarization takes 10^{-11} - 10^{-14} s to build up and is independent of temperature.

Ionic polarization is given by $P_i = N\alpha_i E$

For most materials, the ionic polarisability is less than electronic polarizability. Typically $\alpha_i = \frac{1}{10} \alpha_e$

iii) Orientation or dipole Polarisation

This polarization is a characteristic of polar dielectrics which consists of molecules having permanent dipole moment. In the

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

absence of external electric field, the orientation of dipoles is random resulting in a complete cancellation of each others effect (figure 10).

When the electric field is applied, the molecular dipoles rotate about their axis of symmetry and tend to align with the applied field and the dielectric acquires a net dipole moment and it is orientation polarization.

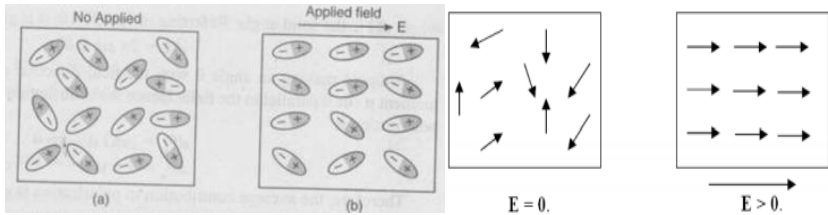


Figure 10

The dipole alignment is counteracted by thermal agitation. Higher the temperature, the greater is the thermal agitation. Hence, orientation polarization is strongly temperature dependent.

In case of solids, the rotation of polar molecules may be highly restricted by the lattice forces, leading to a great reduction in their contribution to orientation polarization. Because of this reason, while the dielectric constant of water is about 80, that for solid ice is only 10.

As the process of orientation polarization involves rotation of molecules, it takes relatively longer time than other two polarisations.

The build up time is of the order of 10^{-10} s or more. The orientation

polarizability $\alpha_0 = \frac{\mu^2}{3kT}$ and orientation polarization $P_0 = \frac{N\mu^2 E}{3kT}$

Orientation polarization is inversely proportional to temperature and proportional to the square of the permanent dipole moment.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

4. Space charge or Interface polarization:

This polarisation occurs in multiphase dielectric materials in which there is a change of resistivity between different phases, when such materials are subjected to an electric field, especially at high temperatures, the charges get accumulated at the interface, because of sudden change in conductivity across the boundary (figure 11). Since the accumulation of charges with opposite faces occurs at opposite parts in the low resistivity phase, in effect it leads to the development of dipole moment within the low resistivity phase domain. Eg. Non homogenous materials such as composites.

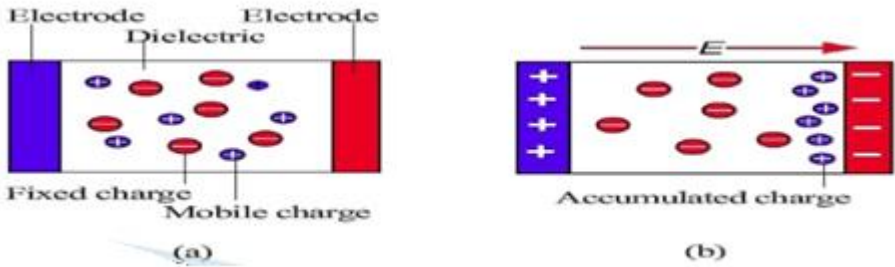


Figure 11

Internal field in a solid for one dimensional infinite array of dipoles

When a dielectric material, either solid or liquid is subjected to an external electric field, each of the atoms develop a dipole moment and acts as an electric dipole. Hence the resultant field at any given atom will be the sum of applied electric field and the electric field due to the surrounding dipoles. The resultant local field is called the internal field (E_i) and is defined as the electric field that acts at a site of any given atom of a solid or liquid dielectric subjected to an external electric field and is the resultant of the applied field (E) and the field due to all the surrounding dipoles (E').

$$E_i = E + E'$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

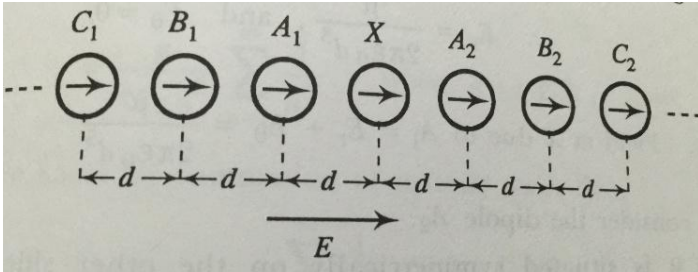


Figure 13: Linear array of atoms in an electric field

The total field at 'X' which is the internal field E_i , is the sum of the applied field (E) and the field due to all the dipoles (E')

$$E_i = E + E' = E + \frac{1.2\mu}{\pi\epsilon_0 d^3}$$

Thus, the combined effect of induced dipoles of neighbouring atoms is to produce a net field at the location of a given atom, which is larger than the applied field.

ClausiusMosotti Equation

Let us consider a solid dielectric, which exhibits electronic polarizability. If α_e is the electronic polarizability per atom, it is related to the bulk polarization P through the relation

$$P = \alpha_e N E_i$$

Therefore $\alpha_e = \frac{P}{N E_i} \dots (1)$

where N is the number of atoms per m^3 and E_i is the local field.

From Lorentz field equation $E_i = E + \frac{P}{3\epsilon_0} \dots (2)$

Substituting equation (2) in equation (1) we get

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$\alpha_e = \frac{P}{N[E + \frac{P}{3\epsilon_0}]}$$

We have $E = \frac{P}{\epsilon_0(\epsilon_r - 1)}$

Substituting for E in the above equation, we obtain

$$\alpha_e = \frac{P}{N[\frac{P}{\epsilon_0(\epsilon_r - 1)} + \frac{P}{3\epsilon_0}]}$$

$$\frac{N\alpha_e}{\epsilon_0} = \frac{1}{[\frac{1}{\epsilon_r - 1} + \frac{1}{3}]} = \frac{1}{[\frac{\epsilon_r + 2}{3(\epsilon_r - 1)}]}$$

$$\frac{3(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{\epsilon_0}$$

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0}$$

The above equation is known as Clausius Mosotti equation.

SOLVED NUMERICALS:

1. What is the probability of a level lying 0.01 eV below the Fermi level not being occupied by electrons at T = 300K?

Solution:

Probability of electrons not being occupied = $1 - f(E)$

$$= 1 - (e^{(E - E_F)/k_B T} + 1)^{-1}$$

$$= 1 - (e^{0.01/0.026} + 1)^{-1}$$

$$= 1 - [1/(1.47 + 1)] = 0.405$$

2. Find the temperature at which there is 1% occupancy probability of a state 0.5 eV above Fermi energy.

Solution:

$f(E) = 0.01 = 1/[e^{(E - E_F)/k_B T} + 1]$ for $E - E_F = 0.5$ eV

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Solving we get $0.01 = 1/[e^{5797/T} + 1]$.

Thus $e^{5797/T} = 1/0.01 - 1 = 99$.

Taking log we get, $T = 5797/4.595 = 1261.1 \text{ K}$

3. The effective mass of holes in a material is 4 times that of electrons. At what temperature would the Fermi energy be shifted by 10% from the middle of the forbidden energy gap? Given band gap = 1 eV.

Solution: $E_F = (E_C + E_V)/2 + (3kT/4) \log(m_h/m_e)$

Fermi level is shifted by 10% = 0.1 eV.

Originally Fermi energy was 0.5 eV above E_V .

Now it is $0.5+0.1 = 0.6 \text{ eV}$ above E_F .

$$(E_V + 0.6) \text{ eV} = (E_C + E_V)/2 + (3kT/4) \log(4) \text{ ----(1)}$$

$$\text{And } (E_V + 0.5) \text{ eV} = (E_C + E_V)/2 \text{ -----(2)}$$

Subtracting 2 from 1 we get $0.1 \text{ eV} = (3kT/4) \log(4)$

$$0.1602 \times 10^{-19} \text{ J} = (3 \times 1.38 \times 10^{-23} \times T/4) 0.6021.$$

$$T = 1116 \text{ K}$$

4. For an intrinsic semiconductor with gap width $E_g = 0.7 \text{ eV}$. Calculate the concentration of intrinsic charge carriers at 300K assuming that $m_e^* = m_o$ (rest mass of electron).

$$n_i = 2 \left[\frac{2\pi m k T}{h^2} \right]^{3/2} e^{-E_g/2kT} \quad n_i = 33.49 \times 10^{18} / \text{m}^3$$

5. The Fermi level in silver is 5.5eV at 0K. Calculate the number of free electrons per unit volume and the probability occupation for electrons with energy 5.6eV in silver at the same temperature.

Solution:

$$E_{F_0} = \left[\frac{h^2}{8m} \right] \left[\frac{3}{\pi} \right]^{2/3} n^{2/3} \quad n = 5.84 \times 10^{28} \text{ m}^{-3}.$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

6. Calculate the probability of an electron occupying an energy level 0.02eV above the Fermi level at 200K and 400K in a material.

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

$$f(E) = \frac{1}{1 + e^{\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 200}}} = 0.24$$

$$f(E) = 0.36 \text{ at } 400\text{K.}$$

7. A semiconducting material 12 mm long, 5 mm wide and 1 mm thick has a magnetic flux density of 0.5 Wb/m² applied perpendicular to the largest faces. A current of 20 mA flows through the length of the sample, and the corresponding voltage measured across its width is 37μV. Find the Hall coefficient of the semiconductor.

Solution:

$$\text{Hall coefficient } R_H = E_y / J_x B_z = -1 / ne.$$

$$\text{Since } E_y = V_y / w, R_H = V_y / w J_x B_z$$

$$\text{Thus } R_H = (37 \times 10^{-6} \times 10^{-3}) / (20 \times 10^{-3} \times 0.5) = 3.7 \times 10^{-6} \text{ m}^3 \text{C}^{-1}$$

8. Calculate the dielectric constant of NaCl, if a NaCl crystal is subjected to an electric field of 1000 V/m and the resulting polarization is 4.3x10⁻⁸ C/m².

$$P = \epsilon_0 (\epsilon_r - 1) E$$

$$(\epsilon_r - 1) = \frac{P}{\epsilon_0 E} = \frac{4.3 \times 10^{-8}}{8.856 \times 10^{-12} \times 10^3} = 0.0049$$

$$\epsilon_r = 1.0049$$

9. The dielectric constant of Helium at 0°C is 1.000074. The density of atoms is 2.7x10²⁵/m³. Calculate the dipole moment induced in each atom when the gas is placed in an electric field of 3x10⁴ V/m.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Dipole moment induced in each atom is $\alpha_e E$ where the electronic

$$\text{polarizability } \alpha_e = \frac{\epsilon_0(\epsilon_r - 1)}{N} = \frac{8.854 \times 10^{-12} \times 0.000074}{2.7 \times 10^{25}} = 2.4255 \times 10^{-41}$$

$$\text{Dipole moment} = 2.4255 \times 10^{-41} \times 3 \times 10^4 = 7.2767 \times 10^{-37} \text{ Cm}$$

10. An elemental solid dielectric material has a polarizability $7 \times 10^{-40} \text{ Fm}^2$. Assuming the internal field to be Lorentz field, calculate the dielectric constant for the material if the material has 3×10^{26} atoms/ m^3 .

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha_e}{3\epsilon_0} = \frac{3 \times 10^{26} \times 7 \times 10^{-40}}{3 \times 8.854 \times 10^{-12}} = 0.7906$$

$$(\epsilon_r - 1) = (\epsilon_r + 2) \times 0.7906$$

$$\epsilon_r = \frac{2.5812}{0.2094} = 12.33$$

S. No	Sample Questions
1.	An electron is accelerated by an electric field of 4V/cm, is found to have mobility $8 \times 10^{-3} \text{ m}^2 / \text{Vs}$. What is its drift velocity?
2.	How many valence electrons will a donor impurity has in a n-type semiconductor?
3.	What is a hole in context of semiconductors?
4.	In Hall effect experiment what is the polarity of Hall voltage for a n-type semiconductor?
5.	What will be the Fermi velocity of an electron in copper if Fermi energy (E_F)= 6 eV ?
6.	At 300K, if probability for occupancy of an energy state E by an electron is 0.75, calculate probability for occupancy of the same state by a hole?
7.	Write any two assumptions of Drude-Lorentz theory?
8.	Sketch the graph of Fermi factor $f(E)$ verses E for the case

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

	$E=E_F$ at $T > 0K$ in metals.
9.	Define density of states in metals.
10.	Write an expression for density of states in metals.
11.	Sketch the variation of fermi level with temperature for n type semiconductor.
12.	What are Fermions?
13.	Outline the phenomenon of Hall effect in materials.
14.	For silicon semiconductor with band gap $1.12eV$, determine the position of the Fermi level at $300K$ if $m_e^* = 0.12m_0$ and $m_h^* = 0.28m_0$.
15.	Distinguish between intrinsic and extrinsic semiconductors.
16.	Find the probability that energy level at $0.2 eV$ below Fermi level being occupied at temperature $1000K$?
17.	What is the value of Fermi function when $E=E_F$ at $T > 0K$?
18.	What is the effect of increase of impurity concentration on band gap in extrinsic semiconductors?
19.	Mention any two demerits of classical free electron theory.
20.	Find the probability of a level lying $0.01 eV$ below the Fermi level being occupied by electrons at $T = 0K$?
21.	What is the magnitude of Lorentz force in Hall effect experiment?
22.	With neat sketch, show the Fermi level position in p-type semiconductor.
23.	Give the expression for Ohm's law in terms of J, σ and E .
24.	What is Fermi factor in Fermi Dirac distribution?
25.	Find the relaxation time of conduction electrons in a metal if its resistivity is $2.5 \times 10^{-8} \Omega m$ and it has 5×10^{28} conduction electrons/ m^3
26.	Sketch the position of Fermi level at $0K$ in a band diagram of a n-type semiconductor, at low doping
27.	Find the Fermi velocity of conduction electron if the Fermi

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

	energy of silver is $8eV$.
28.	Determine the probability of occupancy of an energy level situated $0.05eV$ above the Fermi energy at temperature of $0K$
29.	Write an expression for carrier concentration of p-type semiconductor.
30.	Write any two postulates of classical free electron theory of solids.
31.	Give the relation between electrical conductivity and mobility of charge carriers in a conductor.
32.	If a system is composed of indistinguishable, half integral spin particles and obeys Pauli exclusion principle, then what statistics is obeyed by the system?
33.	What is the value of the Fermi factor for metals at room temperature?
34.	Plot variation of Fermi factor with temperature in a metal.
35.	Distinguish between free electron theory and band theory of solids in terms of influence of lattice on the electron moving in a metal.
36.	Give expression for Fermi level at $0k$ in an intrinsic semiconductor.
37.	Find the temperature at which there is 1% probability that a state with energy $0.5 eV$ above Fermi energy is occupied?
38.	What is Hall Effect?
39.	Sketch the variation in the energy of the Fermi level in a 'n' type semi-conductor as a function of temperature?
40.	Describe in words Wiedemann-Franz Law
41.	What is the formula for intrinsic carrier density (n_i)?
42.	A wire of diameter 0.2 meter contains 10^{28} free electrons per cubic meter. For an electric current of $10A$, calculate the drift velocity for free electrons in the wire?
43.	The fermi level in an intrinsic semi-conductor is at $.25 eV$.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

	What is the width of the band gap?
44.	Electrical conductivity of Cu is $5.78 \times 10^7 \Omega^{-1} \text{m}^{-1}$. If the free electron density of Cu is $8.46 \times 10^{28} \text{m}^{-3}$. Find the mobility of electrons?
45.	The fermi energy for an intrinsic semiconductor is at 5 eV. At 0K, calculate the probability of occupation of electrons at $E = 5.5 \text{eV}$?
46.	A sample of silicon is doped with 10^7 phosphorous atoms/ cm^3 . Find the Hall voltage, if the sample is $100 \mu\text{m}$ thick, $I_x = 1 \text{mA}$ and $B_z = 10^{-5} \text{Wb/m}^2$?
47.	Write any one drawback of classical free electron theory?
48.	Write the relation for specific heat of a metal as per quantum free electron theory
49.	Write the condition at which the value of $f(E) = 1$ at 0 K.
50.	Mention any two assumptions of quantum free electron theory.
51.	Find the relaxation time of conduction electrons in a metal of resistivity $1.54 \times 10^{-8} \Omega \text{m}$. If the metal has 5.8×10^{28} conduction electrons per m^3 .
52.	Find the probability with which an energy level 0.02 eV below Fermi level will be occupied at room temperature of 300K.
53.	A copper strip of 2.0 cm wide and 1.0 mm thick is placed in a magnetic field of 15000 gauss. If a current of 200 A is setup in the strip with the Hall voltage appears across the strip is found to be 0.18 V. calculate the Hall coefficient.
54.	Which statistical rule is obeyed by electrons in quantum free electron theory?
55.	Where does the Fermi level lie in case of n type semiconductor with high impurity concentration?
56.	Electron concentration in a semiconductor is 10^{20}m^{-3} .

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

	Calculate Hall coefficient?
57.	What is doping in semiconductors?
58.	Evaluate the probability of occupation of an energy level 0.4 eV below the Fermi energy level in metal at zero Kelvin.
59.	Copper has electrical conductivity of $9 \times 10^7 \Omega^{-1} \text{m}^{-1}$ and thermal conductivity of $300 \text{ Wm}^{-1} \text{K}^{-1}$ at 305K. Find the Lorentz's number on the basis of classical free electron theory?
60.	If the probability of absence of electron in an energy level of valance band of semiconductor is 0.65 what is the probability of occupation in the same level by a hole?
61.	In the band diagram of a p-type semiconductor show the position of the Fermi level when the doping concentration is low?
62.	Graphically show the variation of $\ln(n_e)$ with increasing temperature in Kelvin where n_e is the electron concentration in an intrinsic semiconductor.
63.	A wire of 4 mm radius carries a current of 8A. Find the current density?
64.	Write the postulates of classical free electron theory and explain the failures of classical free electron theory.
65.	Write the success of Quantum free electron theory.
66.	Explain the variation of Fermi factor in metals with temperature.
67.	Explain Fermi Dirac distribution function. Show that at temperatures above 0K probability of occupancy of Fermi level in metals is 50%.
68.	Derive an expression for the electron concentration in metals at 0K.
69.	Derive an expression for the electron concentration in intrinsic semiconductor.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

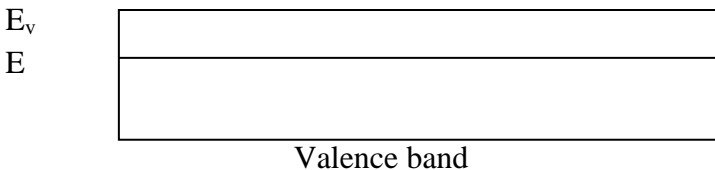
70.	Show that Fermi level of an intrinsic semiconductor lies in the middle of the band gap.
71.	With a neat sketch explain the variation of Fermi level in an n-type semiconductor with the increase in temperature.
72.	What is Hall Effect? Arrive at an expressions for Hall voltage in an n-type semiconductor.
73.	Explain the change of band gap with variation in the doping concentration of an n type semiconductor.
74.	Explain the variation the carrier concentration with increase in temperature in an n type semiconductor.
75.	Define polarization? Explain the four types of polarization.
76.	Arrive at ClausiusMosotti Equation.
77.	Define dielectric polarizability and dielectric susceptibility.

Appendix

Expression for hole concentration in valence band

If $f(E)$ is the probability for occupancy of an energy state at E by an electron, then probability that energy state is vacant is given by $[1 - f(E)]$. Since hole represents the unoccupied state in valence band, the probability for occupancy of state at E by a hole is equal to probability of absence of electron at that level. The hole concentration in valence band is therefore given by

$$p = \int_{\text{bottomband}}^{E_v} [1 - f(E)] g_v(E) dE \quad (7)$$



UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$1-f(E)$ rapidly approaches to zero for lower energy levels, the above equation rewritten as

$$p = \int_{-\infty}^{E_v} [1-f(E)] g_v(E) dE$$

$$p = \int_{-\infty}^{E_v} [1-f(E)] \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE$$

Now $1-f(E) = 1 - \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{e^{\frac{E-E_F}{kT}}}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} \quad (8)$

For $E < E_F$ $(E-E_F)$ is negative. Therefore $e^{\frac{E-E_F}{kT}} \cong 0$

Therefore $1 + e^{\left(\frac{E-E_F}{kT}\right)} \cong 1$

and equation 8 reduces to $1-f(E) = e^{\frac{-(E_F-E)}{kT}}$

$$p = \int_{-\infty}^E \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} e^{\frac{-(E_F-E)}{kT}} dE$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{-\left(\frac{E_F-E_v}{kT}\right)} \int_{-\infty}^{E_v} (E_v - E)^{\frac{1}{2}} e^{-\left(\frac{E_v-E}{kT}\right)} dE$$

Let $E_v - E = x$ then $-dE = dx$ or $dE = -dx$

Lower Lt *Upper Lt*

$x = E_v - (-\infty) \quad E = E_v$

$x = E_v + \infty \quad x = E_v - E_v = 0$

$x = \infty \quad x = 0$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{-\left(\frac{E_F-E_v}{kT}\right)} \int_{\infty}^0 (E_v - E)^{\frac{1}{2}} e^{-\left(\frac{E_v-E}{kT}\right)} (-dE)$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{-\left(\frac{E_F - E_v}{kT}\right)} \int_0^{\infty} (E_v - E)^{\frac{1}{2}} e^{-\left(\frac{E_v - E}{kT}\right)} dE$$

Above equation is of the standard form $\int_0^{\infty} x^{\frac{1}{2}} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$ where

$$E_v - E = x \quad \text{and} \quad a = \frac{1}{kT}$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{\frac{3}{2}} e^{-\left(\frac{E_F - E_v}{kT}\right)} \frac{\sqrt{\pi}}{2} (kT)^{\frac{3}{2}}$$

$$p = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{\frac{3}{2}} e^{-\frac{(E_F - E_v)}{kT}}$$

$$\text{Let } N_v = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

where N_v is temperature-dependent material constant known as effective density of states in the valence band.

$$p = N_v e^{-\frac{(E_F - E_v)}{kT}}$$

Expression for the band gap of a Semiconductor:

The band gap is the energy separation between the conduction band and the valence band of a semiconducting material.

The conductivity of an intrinsic semiconductor is given by

$$\sigma = n_i e \mu_e + n_i e \mu_h$$

$$\sigma = n_i e (\mu_e + \mu_h)$$

Substituting the value of n_i , we get

$$\sigma = 2 \left[\frac{2\pi k T m}{h^2} \right]^{\frac{3}{2}} x \left[\frac{m_e^* m_h^*}{m^2} \right]^{\frac{3}{4}} \exp \left[\frac{-E_g}{2kT} \right] e (\mu_e + \mu_h)$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

The above equation can be written as $\sigma = A \exp\left[\frac{-E_g}{2kT}\right]$

Where $A = 2 \left[\frac{2\pi k T m}{h^2} \right]^{\frac{3}{2}} \times \left[\frac{m_e^* m_h^*}{m^2} \right]^{\frac{3}{4}} e^{(\mu_e + \mu_h)}$

As $\sigma = \frac{1}{\rho} : \rho = B \exp\left[\frac{E_g}{2kT}\right]$ We know that $\rho = \frac{RA}{l}$ then

$$R = B \frac{l}{A} \exp\left[\frac{E_g}{2kT}\right]$$

$$R = C \exp\left[\frac{E_g}{2kT}\right] \quad \text{Where } C = \frac{Bl}{A}$$

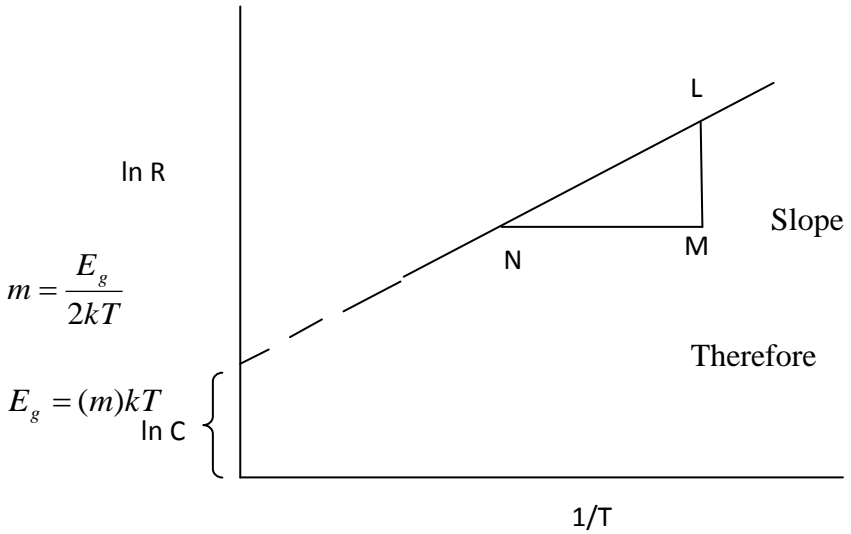
Taking log on both sides $\ln R = \ln C + \frac{E_g}{2kT}$

$$\text{Therefore } \frac{E_g}{2kT} = (\ln R - \ln C)$$

The band gap is given by $E_g = 2kT(\ln R - \ln C)$

$\ln R = \ln C + \frac{E_g}{2kT}$ is of the form $y = mx + c$: By taking $\ln R$ in the y-axis and $\frac{1}{T}$ in the x-axis, if a graph is plotted, a straight line is obtained as shown in below figure.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS



By finding the slope of the straight line, the band gap of the semiconductor is determined using the relation, $E_g = 2k \times \text{slope of the straight line drawn between } \ln R \text{ and } 1/T$.

Lorentz field

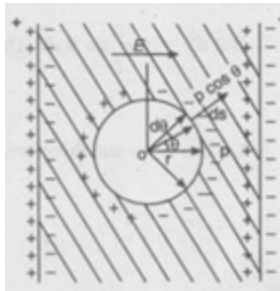


Figure 16

The local field in a three dimensional solid is determined by the structure of the solid.

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

Let us consider a dielectric slab kept in a uniform electric field, E (Figure 15). Let a molecule be at the point O and is surrounded by a spherical cavity of radius r . Let r be arbitrary but sufficiently large compared to molecular dimensions and sufficiently small compared to the dimensions of the dielectric slab. The spherical cavity contains many molecules within it. The molecule at O experiences three electric fields acting on it.

- i) The external electric field E .
- ii) The field E_1 due to induced charges on the surface of the spherical cavity.
- iii) The field E_2 due to the molecular dipoles present in the spherical cavity.

Therefore, the total internal field intensity, E_i is given by

$$E_i = E + E_1 + E_2$$

To calculate E_1 , let us imagine that the dielectric is removed from the sphere. For the actual pattern of the electric field not to be distorted, a surface electric charge should be placed on the spherical surface. At each point of the sphere, the surface charge density is given by

$$\sigma = P \cos \theta$$

where θ is the angle between radius vector \mathbf{r} and the direction \mathbf{E} . The charge on the element dS of the surface of the sphere will be

$$dq = \sigma dS = P \cos \theta dS$$

This charge will produce electric field intensity dE_1 at the centre of the sphere

$$dE_1 = \frac{dq}{4\pi\epsilon_0 r^2} = \frac{P}{4\pi\epsilon_0 r^2} \cos\theta dS$$

This electric field can be resolved into two components: one component $dE_1 \cos \theta$ parallel to the direction of E and the other $dE_1 \sin \theta$ perpendicular to the direction of E .

$$dE_1 \cos \theta = \frac{P}{4\pi\epsilon_0 r^2} \cos^2 \theta dS$$

UNIT- III: ELECTRICAL PROPERTIES OF METALS AND SEMICONDUCTORS

$$dE_1 \cos\theta = \frac{P}{4\pi\epsilon_0 r^2} \cos\theta \sin\theta dS$$

It is obvious that the perpendicular components of the upper and lower half of the sphere cancel each other and only the parallel components contribute to the total intensity E_1 . E_1 is obtained by integrating dE_1 over the whole surface area of the sphere. Thus,

$$E_1 = \int_0^\pi dE_1 \cos\theta dS = \frac{P}{4\pi\epsilon_0 r^2} \int_0^\pi \cos^2\theta dS$$

But $dS = 2\pi r^2 \sin\theta d\theta$. Therefore,

$$E_1 = \frac{P}{2\epsilon_0} \int_0^\pi \cos^2\theta \sin\theta d\theta$$

Let $\cos\theta = x$ and therefore, $-\sin\theta d\theta = dx$.

when $\theta=0$, $\cos\theta=1$ and $\theta=\pi$, $\cos\theta=-1$

$$E_1 = -\frac{P}{2\epsilon_0} \int_1^{-1} x^2 dx = -\frac{P}{2\epsilon_0} \left[\frac{x^3}{3} \right]_1^{-1} = -\frac{P}{2\epsilon_0} \left[\frac{-1}{3} - \frac{1}{3} \right] = \frac{-P}{2\epsilon_0} \left(\frac{-2}{3} \right) = \frac{2P}{6\epsilon_0} = \frac{P}{3\epsilon_0}$$

As there exists symmetrical distribution of molecular dipoles around the molecules at O within the cavity, their contribution cancel each other.

Therefore $E_2=0$.

Hence the total internal field is given by $E_i = E + E_1$

$$E_i = E + \frac{P}{3\epsilon_0}$$

The field given by the above equation is called **Lorentz field** or **local field**.